EQUALLY STRONG HOLES IN A CYLINDRICAL SHELL WITH INITIAL IMPERFECTION

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An isotropic cylindrical shell with initial imperfection of corrugation type is considered, along whose longitudinal section a periodic system of congruent holes with lattice spacing of length H is located. The outline of the j-th hole is denoted by L_j, $L = UL_j$. The direction of the x axis agrees with the direction of the shell axis, the y axis is orthogonal to it, and the origin is in the center of the outline L_0. Tensile forces are applied to the shell and there are no bending moments

$$S_x^\infty = S_1, \quad S_y^\infty = S_2, \quad S_{xy}^\infty = M_x^\infty = M_y^\infty = M_{xy}^\infty = 0, \tag{1}$$

the hole outlines are free from normal and tangential forces, bending moments and transverse forces. The tangential stress on the L_j outlines is assumed constant and subject to determination just as the shape of the hole.

Let ξ, η be an orthogonal curvilinear coordinate system such that the line $\xi = 0$ agrees with the outline L_0. The boundary conditions on L_0 are

$$S_\xi = S_{\xi\eta} = M_\xi = M_{\xi\eta} = Q_\xi = 0,\ S_\eta = S^* = \text{const}. \tag{2}$$

The differential equations for the stress state of an isotropic cylindrical shell with initial imperfection w_0 [2]:

$$DR\Delta^2(w - w_0) + \partial^2 U/\partial x^2 = 0,\ \Delta^2 U - EhR^{-1}\partial^2(w - w_0)/\partial x^2 = 0. \tag{3}$$

Here U is the stress function, w is the deflection, R, h are the shell radius and thickness, $D = Eh^3/12(1-\nu^2)$; E, ν are the elastic modulus and Poisson ratio, and $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

Introducing the complex stress function $\Psi = U/D + i\sqrt{12(1-\nu^2)}\,(w - w_0)/h$, we reduce the system (3) to one differential equation whose general solution can be represented by the contour integral

$$\Psi(z, \bar z) = \int_L \sum_{k=1}^{2} \Gamma_k(z - \tau, \bar z - \bar\tau)\,[g_{2k-1}(t) + i g_{2k}(t)]\,dt, \tag{4}$$

here $z = (y + ix)/a$; $\bar z = (y - ix)/a$, a is the linear dimension, $\tau = \tau(t)$ is the affix of a point on the contour L and $g_k(t)$ are unknown function to be determined from the boundary conditions. The kernels of (4) are presented in [3], their expansions in powers of the parameter $\varepsilon^2 = \sqrt{12(1-\nu^2)}a^2/16Rh$, assumed small have the form

$$\Gamma_k = \Gamma_{k1} + i\Gamma_{k2},\ k = 1, 2,$$
$$\Gamma_{11} = 0.5 + 2\pi^{-1}\varepsilon^2\mathrm{Re}[w(2z\bar z - z^2) - z\bar z] + O(\varepsilon^4),$$
$$\Gamma_{12} = -2\pi^{-1}\omega + \varepsilon^2\,\mathrm{Re}(z\bar z - z^2/2) + O(\varepsilon^4),$$

and

$$\Gamma_{21} = \mathrm{Im}[-z + \pi\varepsilon^2((2z^3/3 + 6z\bar z^2)\omega + 4z^2\bar z)] + O(\varepsilon^4),$$
$$\Gamma_{22} = \mathrm{Im}[4\pi^{-1}z\omega + \varepsilon^2(z^2/6 + 3\bar z z^2/2)] + O(\varepsilon^4),\ \omega = 0.5\ln z\bar z + \text{const}.$$

The additional component $U^\infty(z, \bar z) = 0.25a^2\mathrm{Re}[(S_1 + S_2)z\bar z + (S_1 - S_2 + 2iS_{12})z^2]$. that takes account of condition (1) should be introduced into (4). The force, moment, and transverse force components in ξ, η coordinates are expressed in terms of the stress function and the deflection according to the formulas

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$$
\begin{gathered}
S_\xi + S_\eta = 4a^{-2}\partial^2 U/\partial z\partial\bar{z}, \\
S_\eta - S_\xi + 2iS_{\xi\eta} = 4a^{-2}e^{2i\theta}\partial^2 U/\partial z^2, \\
M_\xi + M_\eta = -4a^{-2}(1+\nu)D\partial^2 w/\partial z\partial\bar{z}, \\
M_\eta - M_\xi - 2iM_{\xi\eta} = 4a^{-2}e^{2i\theta}(1-\nu)D\partial^2 w/\partial z^2, \\
Q_\xi + iQ_\eta = 8ia^{-3}e^{i\theta}D\partial^3 w/\partial z^2\partial\bar{z},
\end{gathered} \tag{5}
$$

where θ is the angle between the directions of the y axis and the tangent to the coordinate line ξ on its increasing side.

Separating into real and imaginary parts in (4), we subject them to the boundary conditions (2) according to (5). We obtain a system of singular integral equations in the unknown functions $g_k(t)$. We set $g_k(t) = \sum_{j=0}^{\infty} \varepsilon^{2j} g_{kj}(t)$, $\delta = D/a^2$ and introduce the functions

$$
\begin{gathered}
\varphi_{kj}(z) = \frac{2\delta}{\pi i}\int_L \left[\frac{ig_{kj}(t) - \bar{\tau} g_{k+2,j}(t)}{z-\tau} - \ln(z-\tau)\, g_{k+2,j}(t)\right] dt, \\
\varphi_{k+2,j}(z) = \frac{2\delta}{\pi i}\int_L \frac{g_{k+2,j}(t)}{z-\tau}\, dt, \quad \Phi(z) = \varphi'(z), \quad k = 1, 2; \quad j = 0, 1, 2, \ldots, \\
\Phi_{21}^0(z) = \delta\int_L [i(z - \tau - 3\bar{\tau})\, g_{40}(t) + g_{20}(t)]\, dt, \\
\Phi_{41}^0(z) = \delta\int_L [3i(\tau - z)\, g_{40}(t) - g_{20}(t)]\, dt.
\end{gathered}
$$

Let the function $z = \omega(\zeta) = \sum_{j=0}^{\infty} \varepsilon^{2j}\omega_j(\zeta)$ execute a conformal mapping of the exterior of the multiconnected contour L into the exterior of the periodic system of circles $\gamma = \{|\zeta - nH| = 1,\ n = 0, \pm 1, \pm 2, \ldots\}$. Then by retaining components in ϵ^0 and ϵ^2 we convert the system of singular integral equations into the following boundary-value problem of complex variable function theory

$$
S_1 + S_2 + 4\operatorname{Re}\Phi_{40}(z) = S_0^*; \tag{6}
$$

$$
\frac{\zeta^2\omega_0'}{\overline{\omega_0'}}\left[S_1 - S_2 + 2\left(\frac{\overline{\omega_0}}{\omega_0'}\Phi_{40}'(\zeta) + \Phi_{20}(\zeta)\right)\right] = S_0^*; \tag{7}
$$

$$
4\operatorname{Re}\left[\Phi_{41}(z) + \Phi_{41}^0(z)\right] + 4\delta\left[2\sqrt{2(1-\nu^2)}\, w_0(z,\bar{z})/h - 2\operatorname{Re}\varphi_{30}^*(z)\right] = S_1^*; \tag{8}
$$

$$
\begin{gathered}
\frac{\zeta^2}{\overline{\omega_0'}}\Big\{2\Big[\overline{\omega_0}\left(\Phi_{41}'(\zeta) + \Phi_{41}^0(\zeta)\right) + \omega_0'\left(\Phi_{21}(\zeta) + \Phi_{21}^0(\zeta)\right) + \\
+\left(\omega_1 - \frac{\omega_1'\overline{\omega_0}}{\overline{\omega_0'}}\right)\Phi_{40}'(\zeta) + \left(\omega_1' - \frac{\omega_0'\overline{\omega_1'}}{\overline{\omega_0'}}\right)\left(\Phi_{20}(\zeta) + \frac{S_1 - S_2}{2}\right) + 2\delta\omega_0'\left(2\overline{\varphi_{30}^*(\zeta)} - \sqrt{12(1-\nu^2)}\, w_0(z,\bar{z})/h\right\} = S_1^*;
\end{gathered} \tag{9}
$$

$$
\varphi_{30}(z) + z\overline{\varphi_{30}'(z)} + \overline{\varphi_{10}(z)} = h^{-1}\sqrt{12(1-\nu^2)}\partial^2 w_0/\partial\bar{z}^2, \tag{10}
$$

where $S^* = S_0^* + \varepsilon^2 S_1^* + O(\varepsilon^4)$; $\omega_j = w_j(\zeta)$; $\varphi_{30}^*(z) = \int_{z_0}^{z}\varphi_{30}(z)\, dz$; and the notation $\Phi(\zeta)$ is obtained for $\Phi(\omega(\zeta))$.

We obtain a solution of the problem of a periodic system of equally strong holes in a plate from (6) and (7), we find the correction factors for ϵ^2 that take account of shell curvature from (8) and (9), and the functions φ_{10}, φ_{30} associated with the influence of the initial imperfection from (10). The derivation of (10) repeats the reasoning of [4] in the study of plate bending.

Let us use the method of [5] to solve problems of the type (6)-(10) for periodic systems of holes. If the function $F_0(z)$ is defined in the strip $|\text{Im}\ z| < H$ of the complex z plane, then the function F(z) that has the period Hi and agrees with $F_0(z)$ in this strip can be represented in the form

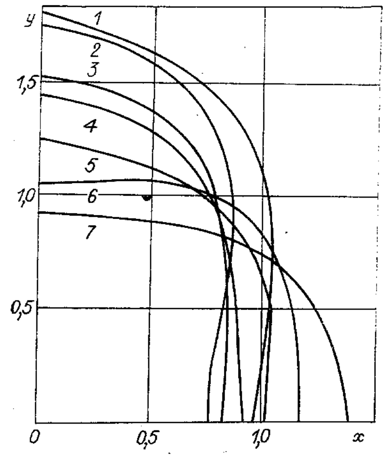

Fig. 1

$$F(z) = F_0(z) + \frac{1}{\pi i} \sum_{k=0}^{\infty} (-1)^k \alpha_{2k+2} \lambda^{2k+2} \int_{L_0} F(\xi)(\xi - z)^{2k+1} d\xi,$$

$$\alpha_{2k} = \sum_{n=1}^{\infty} n^{-2k}, \qquad \lambda = 1/H.$$

In the neighborhood of z = 0 let the following expansion hold $F_0(z) = \sum_{m=0}^{\infty} \lambda^{2m} F_{0m}(z)$, $F_{0m}(z) = \sum_{j=1}^{\infty} \beta_{mj} z^{1-2j}$. Then

$$\begin{gathered} F(z) = \sum_{m=-\infty}^{\infty} B_m z^{2m+1}, \\ B_{-m} = \beta_{0m} + \lambda^2 \beta_{1m} + \lambda^4 \beta_{2m} + O(\lambda^6),\ m = 1, 2, \ldots, \\ B_0 = -2\alpha_2 \lambda^2 \beta_{01} - 2\lambda^4(\alpha_2 \beta_{11} - 3\alpha_4 \beta_{02}) + O(\lambda^6), \\ B_1 = 2\alpha_4 \lambda^4 \beta_{01} + O(\lambda^6). \end{gathered} \tag{11}$$

Taking expansions of the form (11) for the unknown functions, substituting them into (6)-(10) and equating the coefficients of identical powers of λ, we obtain a system of nonlinear algebraic equations whose numerical solution is performed by successive approximations.

The shape of the equally strong hole is determined by the equations [6] $r = |\omega(e^{i\theta})|$.

An estimate of the stiffness of a shell with holes can be performed by determining the reduced parameters. Let the strain energy of a linear element along the line of centers from the hole edge to the middle of a segment between the centers of this and an adjacent hole $Э = \int_{R_0}^{H/2} S_x U_x dx$ in the perforated shell with elastic constants E, ν equal the strain energy $Э_0$ of a linear element of length H/2 along the same line in a solid shell whose elastic constants are E_0, ν_0. We call the constants E_0 the reduced elastic modulus and Poisson ratio and the ratio $k_{re} = \frac{E_0/(1-\nu_0)}{E/(1+\nu)}$ the reduction factor. Using the representation of the stress and displacement components in terms of the functions φ_2, φ_4 and their derivatives $S_x = \mathrm{Re}(2\Phi_4 + z\Phi_4' + \Phi_2)$, $2\mu u_x = \mathrm{Im}(\varkappa\varphi_4 - \bar{z}\overline{\varphi_4'} - \overline{\varphi_2})$, $2\mu = E/(1+\nu)$, $\varkappa = (3-\nu)/(1+\nu)$, we have

$$Э = T/2\mu, \quad T = [Q_{-1}^2 - Q_{-2}(C_1 - 3C_2)](2H^{-2} - 0.25R_0^{-2}) -$$

$$- Q_{-1}(C_1 - C_2)\ln(0.5H/R_0) + C_1 C_2 (H^2/8 - R_0^2) - Q_1(C_1 + 3C_2)(H^4/64 - R_0^4/4).$$

Here $C_1 = S_1 - 0.5\pi\varepsilon^2 D_{-1} + Q_0$; $C_2 = C_1 - \nu(S_1 + S_2 + 2\pi\varepsilon^2 D_{-1})/(1+\nu)$; $Q_j = D_j + \varepsilon^2 P_j$ ($j = 0, \pm 1, \pm 2 \ldots$); D_j, P_j are coefficients of the expansion of the functions $\varphi_{20}(z)$, $\varphi_{21}(z)$ into series of the form (11). If we set $D_j = P_j = R_0 = 0$ and introduce E_0, ν_0 in place of E, ν, then $Э_0 = T_0/2\mu_0$, where $T_0 = S_1(S_1 - \nu_2 S_2)H^2/8(1+\nu_0)$, $2\mu_0 = E_0/(1+\nu_0)$. From the equality $Э = Э_0$ follows $k_{re} = T_0/T$.

TABLE 1

S	F							
	0		0,2		0,4		0,6	
	σ	k_{re}	σ	k_{re}	σ	k_{re}	σ	k_{re}
1,0	3,26	0,816	3,41	0,843	3,65	0,887	3,69	0,963
0,8	2,92	0,843	3,06	0,853	3,20	0,874	3,34	0,912
0,6	2,55	0,906	2,68	0,904	2,82	0,910	2,96	0,929
0,4	2,12	1,009	2,25	0,992	2,38	0,984	2,51	0,988

Calculations were performed for the case when the initial imperfection is given by the function $w_0 = f_0 \cos(2\pi ax/H + cy/R)$, simulating a cylindrical shell with a spiral (for c = a) or cyclic (for c = 0) corrugation having the amplitude f_0 the slope $H/2\pi R$ and the same spacing H as the perforation spacing. It is assumed in the calculations that the Poisson ratios ν, ν_0 of the initial and reduced media are identical and equal to 0.3.

Shown in the figure is the shape of the equally strong hole in a shell with a cyclic corrugation for different values of the loading parameters $S = S_2/S_1$, the lattice $\lambda = a/H$, the corrugation depth $F = f_0Eh/8RS_1$ and the shell curvature ϵ^2. For the curves 2-7 $\epsilon^2 = 0.2$ and correspondingly $S = 1;, 1; 1; 0.6; 0.6; 0.6$, $\lambda = 0.2; 0.1; 0.1; 0.2; 0.1; 0.1$, $F = 0.4; 0.4; 0; 0.4; 0.4; 0$. For the curve 1 $S = 1$, $\lambda = 0.1$, $F = 0$, $\epsilon^2 = 0.4$. The table illustrates the behavior of the stress concentration factors on the hole outline $\sigma = S_\theta/S_1$ and the reduction as a function of the relationships of the transverse and longitudinal forces and the depth of the corrugation.

The following features of the shape of an equally strong hole are detected. For small F, λ, ϵ^2 the hole is elongated along the shell axis, it is reconstructed as each of these parameters grows, and turns out to be extended along the cross section. For instance, for $F = 0$, $\lambda = 0.1$, $\epsilon^2 = 0.2$ the ratio between the length of the transverse radius and the length of the longitudinal equals 0.3, 0.65, 1.5, respectively for S = 0.4, 0.6, 1. The stress concentration factor σ grows as F, ϵ^2, S increase and is practically invariant as λ changes. As F grows the reduction factor k_{re} increases for $S \geq 0.6$ and decreases for lesser values of S. For constant F it diminishes as S and ϵ^2 grow.

In the case of the spiral corrugation, the hole is rotated in the direction of initial shell deflection. However, this rotation is insignificant. The greatest deviation in the values of the radii having identical slope α to the shell axis is achieved for $\alpha = \pm\pi/6$. For example, $[r(-\pi/6) - r(\pi/6)]/r(0)$ for $F = 0.4$, $S = 1 \sim 0.8$, $\varepsilon^2 = 0 \sim 0.4$ and the ratio between the lattice spacing and the shell radius equal to 0.5 fluctuates between 3.5 and 7% limits.

The calculation results lose fidelity for $\epsilon^2 > 0.5$, $\lambda > 0.4$, $F > 5$, $S < 0.3$. The first three constraints are associated with the features of the small parameter methods used in solving the problem, and the last with the impossibility of the existence of equally strong shape of a hole for small S.

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